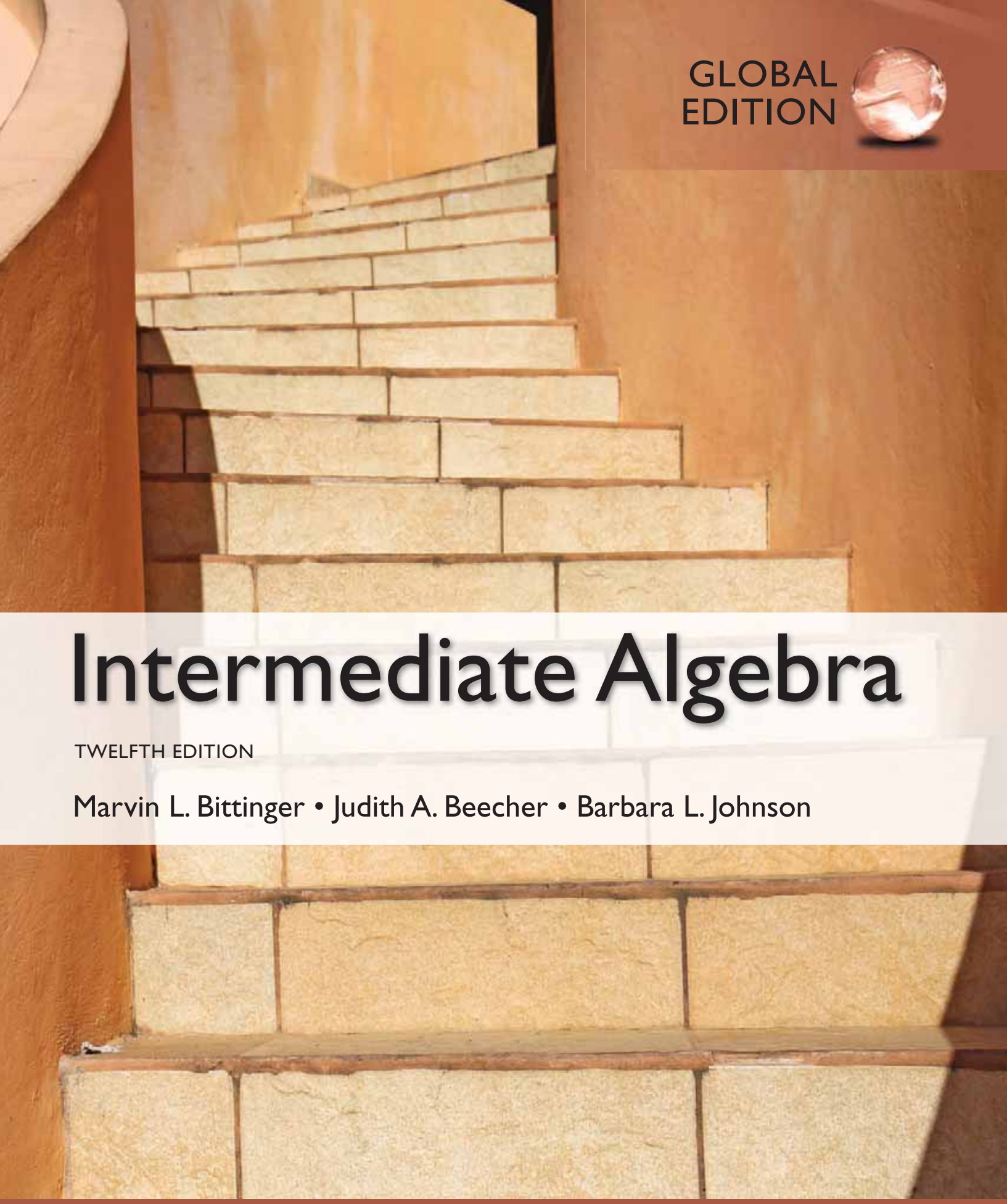

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Intermediate Algebra

TWELFTH EDITION

Marvin L. Bittinger • Judith A. Beecher • Barbara L. Johnson

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PEARSON

Intermediate Algebra

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Contents

Index of Applications 7
Preface 11

R Review of Basic Algebra 21

Part 1 Operations
R.1 The Set of Real Numbers 22
R.2 Operations with Real Numbers 31
R.3 Exponential Notation and Order of Operations 42

Part 2 Manipulations
R.4 Introduction to Algebraic Expressions 51
R.5 Equivalent Algebraic Expressions 57
R.6 Simplifying Algebraic Expressions 65
R.7 Properties of Exponents and Scientific Notation 73

Summary and Review 85
Test 89

1 Solving Linear Equations and Inequalities 91

1.1 Solving Equations 92
1.2 Formulas and Applications 106
1.3 Applications and Problem Solving 115

Mid-Chapter Review 128

1.4 Sets, Inequalities, and Interval Notation 130

Translating for Success 139

1.5 Intersections, Unions, and Compound Inequalities 146
1.6 Absolute-Value Equations and Inequalities 158

Summary and Review 169
Test 175

2 Graphs, Functions, and Applications 177

2.1 Graphs of Equations 178
2.2 Functions and Graphs 192
2.3 Finding Domain and Range 206

Mid-Chapter Review 211

2.4 Linear Functions: Graphs and Slope 213
2.5 More on Graphing Linear Equations 224

Visualizing for Success 231

2.6 Finding Equations of Lines; Applications 236

Summary and Review 247
Test 256
Cumulative Review 259

3 Systems of Equations 261

3.1 Systems of Equations in Two Variables 262
3.2 Solving by Substitution 271
3.3 Solving by Elimination 277
3.4 Solving Applied Problems: Two Equations 286

Translating for Success 294
Mid-Chapter Review 299

3.5 Systems of Equations in Three Variables 301
3.6 Solving Applied Problems: Three Equations 308
3.7 Systems of Inequalities in Two Variables 315

Visualizing for Success 323
Summary and Review 328
Test 335
Cumulative Review 337

4 Polynomials and Polynomial Functions 339

4.1 Introduction to Polynomials and Polynomial Functions 340
4.2 Multiplication of Polynomials 351
4.3 Introduction to Factoring 362
4.4 Factoring Trinomials: $x^2 + bx + c$ 368

Mid-Chapter Review 374

4.5 Factoring Trinomials: $ax^2 + bx + c, a \neq 1$ 376
4.6 Special Factoring 384

Visualizing for Success 390

4.7 Factoring: A General Strategy 395
4.8 Applications of Polynomial Equations and Functions 400

Translating for Success 407
Summary and Review 413
Test 419
Cumulative Review 421

5 Rational Expressions, Equations, and Functions 423

5.1 Rational Expressions and Functions: Multiplying, Dividing, and Simplifying 424
5.2 LCMs, LCDs, Addition, and Subtraction 436
5.3 Division of Polynomials 447
5.4 Complex Rational Expressions 455

Mid-Chapter Review 462

5.5 Solving Rational Equations 464
5.6 Applications and Proportions 472

Translating for Success 480

5.7 Formulas and Applications 486
5.8 Variation and Applications 490

Summary and Review 501
Test 507
Cumulative Review 509

6 Radical Expressions, Equations, and Functions 511

6.1 Radical Expressions and Functions 512
6.2 Rational Numbers as Exponents 523
6.3 Simplifying Radical Expressions 530
6.4 Addition, Subtraction, and More Multiplication 539

Mid-Chapter Review 545

6.5 More on Division of Radical Expressions 547
6.6 Solving Radical Equations 552
6.7 Applications Involving Powers and Roots 563

Translating for Success 565

6.8 The Complex Numbers 570

Summary and Review 581
Test 587
Cumulative Review 589

7 Quadratic Equations and Functions 591

7.1 The Basics of Solving Quadratic Equations 592
7.2 The Quadratic Formula 606

7.3 Applications Involving Quadratic Equations 613

Translating for Success 619

7.4 More on Quadratic Equations 625

Mid-Chapter Review 634

7.5 Graphing $f(x) = a(x - h)^2 + k$ 636

7.6 Graphing $f(x) = ax^2 + bx + c$ 645

Visualizing for Success 650

7.7 Mathematical Modeling with Quadratic Functions 654

7.8 Polynomial Inequalities and Rational Inequalities 665

Summary and Review 673

Test 679

Cumulative Review 681

8 Exponential Functions and Logarithmic Functions 683

8.1 Exponential Functions 684

8.2 Composite Functions and Inverse Functions 698

8.3 Logarithmic Functions 715

8.4 Properties of Logarithmic Functions 726

Mid-Chapter Review 732

8.5 Natural Logarithmic Functions 734

Visualizing for Success 739

8.6 Solving Exponential Equations and Logarithmic Equations 743

8.7 Mathematical Modeling with Exponential Functions and Logarithmic Functions 750

Translating for Success 758

Summary and Review 764

Test 772

Cumulative Review 775

9 Conic Sections 777

9.1 Parabolas and Circles 778

9.2 Ellipses 791

Mid-Chapter Review 799

9.3 Hyperbolas 801

Visualizing for Success 805

9.4 Nonlinear Systems of Equations 809

Summary and Review 819

Test 825

Cumulative Review 827

Appendixes 833

A Fraction Notation 834

B Determinants and Cramer's Rule 843

C Elimination Using Matrices 848

D The Algebra of Functions 852

Answers A-1

Guided Solutions A-43

Glossary G-1

Index I-1

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Index of Applications

Agriculture
Composting, 662
Corral design, 661
Farmers' markets, 242–243
Feed lot, 620
Fenced-in land, 661
Filling a grain bin, 590
Flower bed, 411, 620, 824
Flower bulbs, 296
Garden design, 662, 818
Grain flow, 758
Harvesting time, 590
Livestock feed, 296
Mixing fertilizers, 290–291
Vegetable garden, 824
Vegetable seeds, 296

Astronomy
Alpha Centauri, 88
Brightest star in the sky, 84
Distance from Earth to Mars, 79
Distance from the earth to the sun, 78
Distance from the sun to the planet Saturn, 77
Light from the sun to Pluto, 79
Lunar eclipses, 486–487
Mass of Jupiter, 79
Mass of Pluto, 90
Orbit of Venus, 84
Planetary orbit, 795
Satellite's escape velocity, 489
Weight of an astronaut, 500
Weight on Mars, 483, 498
Weight on moon, 483

Automotive
Alternative fueling stations, 696
Auto dealers, 246
Automobile pricing, 312
Automotive maintenance, 297, 298
Automotive repair, 585
Daytime accidents, 663
Fuel economy, 298
Gas mileage, 475, 485
Horsepower of an engine, 114
Hybrid vehicle models sold in the United States, 684
Median age of cars, 256
Models of Indy Cars, 482
Nighttime accidents, 663
Plug-in vehicle sales, 758
Speed of a skidding car, 561
Stopping distance of a car, 499, 662

Biology
Atoms in the human body, 84
Bacteria growth, 697, 771
Bees and honey, 492
Blue whale, 759
Diameter of a helium atom, 77
Estimating fish population, 480, 483
Estimating wildlife population, 476–477, 483
Insect-eating lizard, 83
Mass of an electron, 83
Mass of water in body, 498
Otter population, 760

Business
Artist's prints, 121
Book club, 830
Book sale, 312
Break-even point, 818
Catering, 296, 474–475
Container sizes, 168
Copy budget, 175
Delivering leaflets, 562
Delivery service, 144
Filling sandbags, 472–473
Homespun Jellies, 255
Internet search ads, 126
Inventory, 338
Lens production, 314
Maximizing profit, 662
Median size of a grocery store, 828
Minimizing cost, 661
Newspaper circulation, 221
Office expense, 294
PDQ shopping network, 480
Placing wrappers on canned goods, 481
Printing and engraving, 84
Printing tee shirts, 481
Radio advertising, 260, 338
Renting office space, 144, 153
Retail and advertising, 771
Rice production, 115–116
Sales, 245, 287, 295, 312, 333, 335, 407, 658, 762, 818
Shipping, 497
Super Bowl commercials, 309–310
Tea Mug Collective, 698
Telemarketing, 314
Ticket profits, 660
Total cost, 347, 366
Total profit, 348, 672
Total revenue, 347, 366
Volume of mail, 221
Wine production, 483

Chemistry
Acid mixtures, 775
Ammonia solutions, 831
Carbon dating, 757, 761, 771, 774, 831
Half-life, 762
Hydrogen ion concentration, 751, 752, 759, 774
Ink remover, 296
Mixing solutions, 291, 300, 335, 338, 407, 633
Orange drink mixtures, 333
pH of liquids, 751, 752, 759
pH of tomatoes, 774
Temperature conversions, 202
Temperatures of liquids, 156
Temperatures of solids, 144

Construction
Architecture, 273–274, 661, 682, 813–814
Beam weight, 498
Box construction, 621
Burj Khalifa in Dubai, 604
Carpentry, 406
Clearing a lot, 482
Covered bridges, 762
Cross section of a roof, 117
Cutting a board, 120
Doorway construction, 790
Fencing, 139
Footer of a house, 564
Gateway Arch, 604
Grade of a stairway, 223
Home construction, 762

Installing seamless guttering, 120
Jackhammer noise, 759
Ladder location, 411, 420, 614, 615
Lot dimensions, 260
Molding plastics, 661
New housing permits, 471
Painting, 481, 505, 562
Pitch of a roof, 222
Plank height, 565
Plumbing, 568, 762
Predicting paint needs, 508
Retaining wall, 484
Roofing, 569
Room dimensions, 175
Rope cutting, 124, 173, 483
Siduhe River Bridge in China, 604
Stained-glass window design, 655
Tokyo Sky Tree in Japan, 604
USS Constitution model, 482
Washington Monument, 604
Wire cutting, 124, 338, 619, 758, 818
Workbench design, 411

Consumer

Aluminum usage, 498
Carpet cleaning, 125
Cell-phone subscribers, 82
Coffee consumption, 483
Cost of a necklace, 241–242
Cost of a service call, 242
Coupon redemptions, 82
First-class postage, 761
Fitness club costs, 245
Free ornamental tree offer, 285
Gasoline prices, 619
Hot dog consumption, 83
Insurance benefits, 144
Insurance claims, 144
Insurance premiums, 119, 126
Luxury purchases in China, 223
Mother's Day spending, 422
Moving costs, 173, 176
Photo prices, 421
Pizza prices, 664
Popcorn prices, 333
Price of auto detailing, 117–118
Price of cross training shoes, 118
Purchase price, 124
Purchasing books and games, 295
Real estate prices, 127
Repair rates, 336
School photos, 125
Taxi fare, 191
Utility cost, 338
Wedding costs, 144

Economics

Average sale price of homes in central Indiana, 178
Average tax refund, 223
Salvage value, 696, 760
Stimulating the hometown economy, 282
Stock prices, 662
Supply and demand, 145, 663
Tax code, 461
Tax revenue, 88
U.S. tax code, 761
World demand for oil, 827

Education

Bachelor's degrees, 223
College enrollment, 220
Community college credits, 285
Exam scores, 399
Female medical school graduates, 129
Forgetting, 775
Full-time equivalent students, 113
Grades, 143
Quiz scores, 139
Reading assignment, 139
SAT scores, 294
Scholastic aptitude test, 311
School enrollment, 262–263
School fundraiser, 245
Student debt, 220
Student loans, 289, 294, 296, 297, 313
Test scores, 338, 506
Tuition and fees at two-year colleges, 145
Tuition cost, 253
Valley Community College, 662, 663

Engineering

Antenna wires, 411
Beam load, 831
Bridge expansion, 567
Conductance, 41
Current and resistance, 498
Design, 276, 411, 661
Distance over water, 568
Guy wire, 567, 619
Height of a tower, 561
Landscape design, 614–615, 654–655
Nuclear energy, 763
Ohm's Law, 491
Power of electric current, 506
Resistance, 487, 496
Road grade, 219
Solar panel support, 116–117
Solar power, 762
Suspension bridge, 659–660
Town planning, 614
TV signal, 496
Wind energy, 347
Wind turbines, 483

Environment

Amazon River water flow, 84
Corona Arch in Moab, Utah, 484
Dewpoint spread, 145
Greenhouse gases, 174
Hurricanes in the Atlantic Basin, 663
Melting snow, 202, 491
Precipitation in Sonoma, California, 658
Recycling aluminum cans, 697
River depth, 664
Slope of a river, 222
Waste generation, 419, 510
Waterfalls, 327
Wind chill temperature, 569
Wind-driven forest fires, 795, 800

Finance

Charitable contributions, 758
Coin value, 489
Compound interest, 360, 690, 691, 692, 695, 696, 733, 734, 752, 828
Counterfeit money, 471
Interest compounded continuously, 753–754, 760, 771, 774
Investment, 56, 144, 173, 289, 296, 297, 300, 308, 310, 313, 330, 336, 692, 771, 818, 826
Making change, 297
Money borrowed, 565
Money remaining after a purchase, 55
Simple interest, 56, 109, 114
Small-business loans, 330

Geometry

Angle measures, 127, 314, 407
Angles of a triangle, 123, 175, 308, 309, 311, 333, 565
Area of a balloon, 508
Area of a circle, 500
Area of a dining table, 56
Area of an envelope, 410
Area of a parallelogram, 56
Area of a rug, 86
Area of a square, 127
Area of a trapezoid, 108, 114
Area of a triangle, 54, 89, 567, 590
Book area, 410
Bookcase width, 586
Carton dimensions, 823
Complementary angles, 276, 284, 407
Diagonal of a rectangle, 564, 567, 568
Diagonal of a square, 565
Dimensions of a rectangular region, 124, 129, 139, 275, 276, 294, 300, 407, 418, 420, 480, 619, 620, 621, 622, 677, 758, 814, 817, 818, 823, 826, 831
Distance between the parallel sides of an octagon, 569
Home office dimensions, 826
Length of a side of a square, 223, 410, 418, 567, 568, 586, 588, 826
Length of a side of a triangle, 223, 406, 567, 569, 619, 621, 622
Lengths of a rectangle, 565
Maximum area, 654–655, 679, 682, 831
Number of diagonals, 366
Perimeter of a hexagon, 682
Perimeter of an octagon, 399
Perimeter of a pentagon, 399
Perimeter of a rectangular region, 69, 565
Perimeters of polygons, 831

Quilt dimensions, 621
Radius of a circular region, 824
Sides of a square, 480
Supplementary angles, 276, 284, 294, 407
Surface area, 350, 624
The Golden Rectangle, 624
Triangle dimensions, 294, 406, 407, 412, 538, 619, 620, 758
Triangular layers, 350, 366
Volume of carpeting, 394
Volume of a plastic sheet, 84
Volume of a propane gas tank, 366
Volume of a tree, 496
Width of the margins of a book, 622

Government

Average age of House members, 201
Average age of senators, 201
Cost of health care in the United States, 774
Supreme Court Justices and appointing president, 193

Health/Medicine

Acidosis, 752
Alkalosis, 759
Audiology, 751
Basal metabolic rate, 112
Body mass index, 106–107, 143, 157
Body surface area, 519
Calorie requirement, 113
Calories burned, 129, 476, 653
Cholesterol levels, 146, 312
Diabetes, 126, 245
Exercise, 156, 298
Exercise danger zone, 327
Fat content of fast food, 311
Fat intake, 497
Food science, 297
Human blood, 483
Ibuprofen, 347
Knee replacements, 696
Lithotripter, 795
Medicine dosage, 113, 157, 653
Nutrition, 312, 313
People with Alzheimer's, 223
Pharmacists, 205
Physical therapists, 137
Prescription coverage, 144
Projecting birth weight, 113
Relative size of a hemoglobin molecule vs. a human egg, 79
Veterinary expenditure, 312
Veterinary medicine, 342–343
Waiting lists for organ transplants, 191

Labor

Adjusted wages, 127
Bonuses, 282
Earnings, 490, 562
Income vs. time, 508
Median amount of pay by year, 51–52
Median income by age, 347
Real estate commission, 118–119, 125, 191
Salary, 125, 138, 139, 144, 260
Sales commission, 139
Wages, 373
Work rate, 498
Working alone, 474–475, 480, 481, 482, 562, 624
Working together, 473, 481, 482, 565, 831

Miscellaneous

Age of Diaphantos, 832
Archaeology, 275
Balloon bouquets, 295
Blending granola, 296
Blending spices, 287–288
Cake servings, 664
Child's block, 411
Chocolate assortments, 295
Chocolate making, 775
Cleaning bleachers, 493
Coffee, 288, 312, 633, 752
Coin mixture, 565, 619, 758
Completing a puzzle, 508
Converting dress sizes, 713
Cutting firewood, 481
Digits, 314
Dishwasher noise, 759
DVD collections, 139
Easter Island, 620
Escalators, 485
Filling a tank, 475, 510, 682
Filling time, 480, 481
Furniture polish, 296
Hat size as a function of head circumference, 243
IRS instruction booklet, 198–199
Jewelry design, 308–309
Landscaping, 191, 613, 619, 621, 775
Locker numbers, 619
Lunch orders, 286–287
Mass of a grain of sand, 78
Mirror framing, 621
Mixing fruit juice, 295
Mixing rice, 373
Most expensive furniture, 763
Mystery numbers, 294
Nontoxic floor wax, 296
Nontoxic scouring powder, 276
Photoreceptor rod, 83
Picture matting and framing, 411, 622, 677
Pizza crusts, 624
Post office box numbers, 125
Pressing shirts, 482
Prize tee shirts, 404–405
Pumping rate, 498
Pumping time, 506
Pyramide du Louvre, 567
Raffle tickets, 314, 621
Safety flares, 412
Seconds in 2000 years, 83
Shoveling time, 565
Sighting to the horizon, 560, 561, 588
Skimming a swimming pool, 482
Sorting donations, 482
Sound of an alarm clock, 759
Speaker placement, 568
Tent design, 410
Trucks moving mulch, 485
Uniform numbers, 758
Value of a lawn mower, 245
Value of a rare stamp, 733
Warning dye, 590
Washing elephants, 481
Watch time, 127
Water flow, 500
Water fountains, 826
Whispering gallery, 795
Word knowledge, 84

Physics

Acoustics, 751
Atmospheric drag, 500
Centripetal force, 831
Combined gas law, 486, 500
Doppler effect, 487
Falling distance, 496, 617–618
Fireworks, 383, 405, 412
Free-falling objects, 604, 679
Height of a rocket, 363
Height of a thrown object, 672
Hooke's Law, 498
Intensity of light, 499
Motion of a spring, 168
Musical pitch, 493, 498
Object's average speed, 489
Pendulums, 538, 562, 616–617
Pressure at sea depth, 156, 176, 202
Sound levels, 750–751, 759, 770
Speed of light, 77
Speed of sound, 558
Temperature and the speed of sound, 561
Temperature as a function of depth, 202
Volume and pressure, 499
Wavelength of a certain red light, 83
Wavelength and frequency, 498
Weight of a sphere, 500

Social Science

Ages, 314
Facebook users, 83
Food stamp program, 126
Foreign adoptions to the United States, 663, 680
Homelessness, 760
Living with grandparents, 204
Ramps for the disabled, 568
Siblings, 298
Social networking, 662
Spread of a rumor, 760

Sports/Entertainment
Art, 763, 817
Baseball diamond, 564
Basketball scoring, 284, 313
Beach volleyball, 613
Bicycling, 477–478, 480, 622, 623
Bungee jumping, 624
Chess ratings, 109–110
Concert series at the Capitol, 558
Diana Nyad swimming from Cuba to Florida, 123
Display of a sports card, 422
Earned run average, 489, 500
Football field, 69
Games in a sports league, 420, 422
Golf, 313, 681
Golf ball stacks, 347
Grade of a treadmill, 220, 222
Hang time, 600–601, 603, 617, 677, 679
Height of a baseball, 366, 383
Height of a softball, 363
Hiking, 484
Hockey, 276, 480, 510
Ironman Triathlon, 123
Kingda Ka roller coaster, 604
Magic number in sports, 348
Marathon times, 604
Marching band performance, 558
NBA court, 124
Rock music sales, 83
Running, 223, 294, 475
Skiing, 622
Snowboarding, 790
Soccer field, 275
Softball diamond, 567
Spending on Summer 2012 Olympic Games, 77
Swimming, 127
Tennis court, 124, 335
Theatrical lighting, 798
Three-point field goals, 482
Ticket revenue, 334
Ticket sales, 662
Touchdown pace, 482
Vertical leap, 601
Walking speed, 759
World running records, 246, 255, 259
Youth football, 416

Statistics/Demographics
Average number of motorcyclists killed, 658
Billionaires, 755–756
Climber deaths, 124
Heaviest pumpkin, 235
Increase in number of residents, 116
Life expectancy, 246, 658
Live births by age, 678
Median age of men at first marriage, 258
Population, 657, 658
Population change, 127
Population decrease, 139, 175, 178, 762
Population growth, 173, 753, 761, 774, 775, 832
World population growth, 761

Technology
Computer calculations, 84
Computer pricing, 312
Computer repair, 255
Computer screens, 563, 818
DVD player screen, 677
Global mobile data traffic, 756
HDTV screens, 814, 818
LCD TVs, 696
Memory board, 679
Optics, 486
Relative aperture, 497
Value of a computer, 223, 245

Transportation
Air travel, 126, 293, 298, 335, 361, 622, 831
Airplane seating, 276
Auto travel, 292
Bicycle travel, 508, 562
Boat travel, 505, 565
Boating, 126, 129, 175, 297, 300, 484, 605
Bus travel, 492
Canoe depth, 664
Canoeing, 297, 622
Car speed, 623, 635
Car travel, 297, 407, 485, 619, 758
Car trips, 622
Cargo area of a van, 817
Chartering a bus, 771
Clearing customs, 124
Commuter travel, 569
Cruising altitude, 126
Distance traveled, 55, 506
Escape ramp on an airliner, 220
Flight into a headwind, 126
Four-wheeler travel, 478
Height of an airplane, 561
Interstate mile markers, 125, 173
Jet travel, 484
Kayaking, 484
Marine travel, 122, 293, 569, 616, 619, 679, 682, 775
Minimizing tolls, 156
Motorcycle travel, 334, 615–616, 677
Moving freight, 461
Moving sidewalks, 122, 173, 485
Navigation, 623
Parking lots, 519, 620, 621
Point of no return, 298
Rapid transit train, 56
Rate of travel, 499
River cruising, 127
Riverboat speed, 479
Road-pavement messages, 564, 566
Sailing, 410, 620, 621
Sightseeing boat, 294
Time and speed, 508
Tour travel, 484
Train speed, 565
Train travel, 292, 297, 333, 338, 758, 776, 830
Transporting by barge, 479
Transporting cargo, 484
U.S. transcontinental railroad, 311

Preface

The Bittinger Program

Math hasn't changed, but students—and the way they learn it—have.

Intermediate Algebra, Twelfth Edition, continues the Bittinger tradition of objective-based, guided learning, while integrating timely updates to the proven pedagogy. In this edition, there is a greater emphasis on guided learning and helping students get the most out of all of the course resources available with the Bittinger program, including new opportunities for mobile learning.

The program has expanded to include these comprehensive new teaching and learning resources: ***MyMathGuide* workbook**, **To-the-Point Objective Videos**, and enhanced, media-rich **MyMathLab** courses. Feedback from instructors and students motivated these and several other significant improvements: a new design to support guided learning, new figures and photos to help students visualize both concepts and applications, and many new and updated real-data applications to bring the math to life.

With so many resources available in so many formats, the trusted guidance of the Bittinger team on *what to do* and *when* will help today's math students stay on task. Students are encouraged to use ***Your Guide to Success in Math***, a four-step learning path and checklist available on the handy reference card in the front of this text and in MyMathLab. The guide will help students identify the resources in the textbook, supplements, and MyMathLab that support *their* learning style, as they develop and retain the skills and conceptual understanding they need to succeed in this and future courses.

In this preface, a look at the key new *and* hallmark resources and features of the *Intermediate Algebra* program—including the textbook/eText, video program, *MyMathGuide* workbook, and MyMathLab—is organized around ***Your Guide to Success in Math***. This will help instructors direct students to the tools and resources that will help them most in a traditional lecture, hybrid, lab-based, or online environment.

NEW AND HALLMARK FEATURES IN RELATION TO Your Guide to Success in Math

STEP 1 Learn the Skills and Concepts

Students have several options for learning, reviewing, and practicing the math concepts and skills.

Textbook/eText

- ☐ **Skill to Review.** At the beginning of nearly every text section, *Skill to Review* offers a just-in-time review of a previously presented skill that relates to the new material in the section. Section and objective references are included for the student's convenience, and two practice exercises are provided for review and reinforcement.
- ☐ **Margin Exercises.** For each objective, problems labeled "Do Exercise . . ." give students frequent opportunities to solve exercises while they learn.

- ☐ ***New!*** **Guided Solutions.** Nearly every section has *Guided Solution* margin exercises with fill-in blanks at key steps in the problem-solving process.
- ☐ ***Enhanced!*** **MyMathLab.** MyMathLab now includes *Active Learning Figures* for directed exploration of concepts; more problem types, including *Reading Checks* and *Guided Solutions*; and new, objective-based videos. (See pp. 14–18 for a detailed description of the features of MyMathLab.)
  - ☐ ***New!*** **Skills Checks.** In the Learning Path for Ready-to-Go MyMathLab, each chapter begins with a brief assessment of students' mastery of the prerequisite skills needed to learn the new material in the chapter. Based on the results of this pretest, a personalized homework set is designed to help each student prepare for the chapter.
- ☐ ***New!*** **To-the-Point Objective Videos.** This is a comprehensive new program of objective-based, interactive videos that are incorporated into the Learning Path in MyMathLab and can be used hand-in-hand with the *MyMathGuide* workbook.
  - ☐ ***New!*** **Interactive Your Turn Exercises.** For each objective in the videos, students solve exercises and receive instant feedback on their work.
- ☐ ***New!*** ***MyMathGuide: Notes, Practice, and Video Path.*** This is an objective-based workbook (available in MyMathLab) for guided, hands-on learning. It offers vocabulary, skill, and concept review—along with problem-solving practice—with space to show work and write notes. Incorporated in the Learning Path in MyMathLab, it can be used together with the To-the-Point Objective Video program, instructor lectures, and the textbook.

STEP 2 Check Your Understanding

Throughout the program, students have frequent opportunities to check their work and confirm that they understand each skill and concept before moving on to the next topic.

- ☐ ***New!*** **Reading Checks.** At the beginning of each set of section exercises in the text, students demonstrate their grasp of the skills and concepts.
- ☐ ***New!*** **Active Learning Figures.** In MyMathLab, Active Learning Figures guide students in exploring math concepts and reinforcing their understanding.
- ☐ **Translating/Visualizing for Success.** In the text and in MyMathLab, these activities offer students extra practice with the important first step of the process for solving applied problems.

STEP 3 Do Your Homework

Intermediate Algebra, Twelfth Edition, has a wealth of proven and updated exercises. Prebuilt assignments are available for instructors in MyMathLab.

- ☐ **Skill Maintenance.** In each section, these exercises offer a thorough review of the math in the preceding text.
- ☐ **Synthesis Exercises.** To help build critical-thinking skills, these section exercises require students to use what they know and combine learning objectives from the current section with those from previous sections.

STEP 4 Review and Test Your Understanding

Students have a variety of resources to check their skills and understanding along the way and to help them prepare for tests.

- ☐ **Mid-Chapter Review.** Midway through each chapter, students work a set of exercises (*Concept Reinforcement, Guided Solutions, Mixed Review,* and *Understanding Through Discussion and Writing*) to confirm that they have grasped the skills and concepts covered in the first half before moving on to new material.
- ☐ **Summary and Review.** This resource provides an in-text opportunity for active learning and review for each chapter. *Vocabulary Reinforcement, Concept Reinforcement,* objective-based *Study Guide* (examples paired with similar exercises), *Review Exercises* (including *Synthesis* problems), and *Understanding Through Discussion and Writing* are included in these comprehensive chapter reviews.
- ☐ **Chapter Test.** Chapter Tests offer students the opportunity for comprehensive review and reinforcement prior to taking their instructor's exam. **Chapter Test-Prep Videos** (in MyMathLab and on YouTube) show step-by-step solutions to the Chapter Tests.
- ☐ **Cumulative Review.** Following every chapter beginning with Chapter 2, a Cumulative Review revisits skills and concepts from all preceding chapters to help students retain previously learned material.

Study Skills

Developing solid time-management, note-taking, test-taking, and other study skills is key to student success in math courses (as well as professionally and personally). Instructors can direct students to related study skills resources as needed.

- ☐ ***New!*** **Student Study Reference.** This pull-out card at the front of the text is perforated, three-hole-punched, and binder-ready for convenient reference. It includes **Your Guide to Success in Math** course checklist, **Student Organizer**, and **At a Glance**, a list of key information and examples for quick reference as students work exercises and review for tests.
- ☐ ***New!*** **Studying for Success.** Checklists of study skills—designed to ensure that students develop the skills they need to succeed in math, school, and life—are integrated throughout the text at the beginning of selected sections.
- ☐ ***New!*** **Study Skills Modules.** In MyMathLab, interactive modules address common areas of weakness, including time-management, test-taking, and note-taking skills. Additional modules support career-readiness.

Learning Math in Context

- ☐ ***New!*** **Applications.** Throughout the text in examples and exercises, real-data applications encourage students to see and interpret the mathematics that appears every day in the world around them. Applications that use real data are drawn from business and economics, life and physical sciences, medicine, technology, and areas of general interest such as sports and daily life. New applications include "Rice Production" (pp. 115–116), "Physical Therapists" (p. 137), "Super Bowl Commercials" (p. 309), "Catering a Business Luncheon " (p. 474), "Beach Volleyball" (p. 613), and "Alternative Fueling Stations" (p. 696). For a complete list of applications, please refer to the Index of Applications (p. 7).

BREAKTHROUGH

To improving results

MyMathLab

Ties the Complete Learning Program Together

MyMathLab® Online Course (access code required)

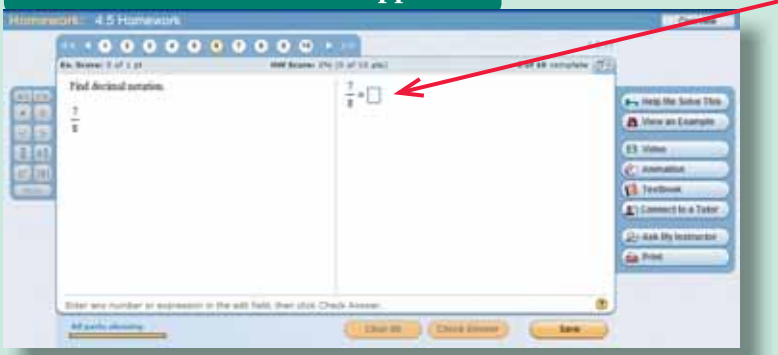
MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy-to-use format. MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future.

MyMathLab for Developmental Mathematics

Prepared to go wherever you want to take your students.

Personalized Support for Students

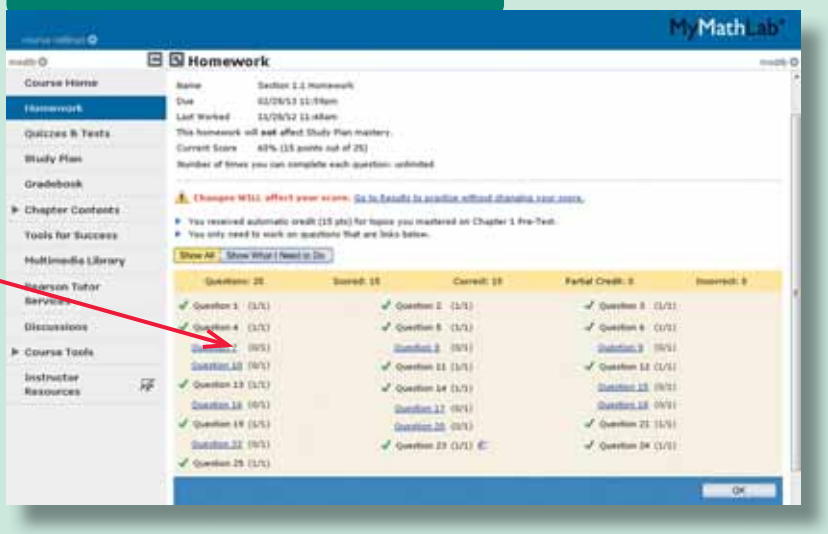
Homework with Built-in Support



Exercises: The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunities for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.

Multimedia Learning Aids: Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at point of use.

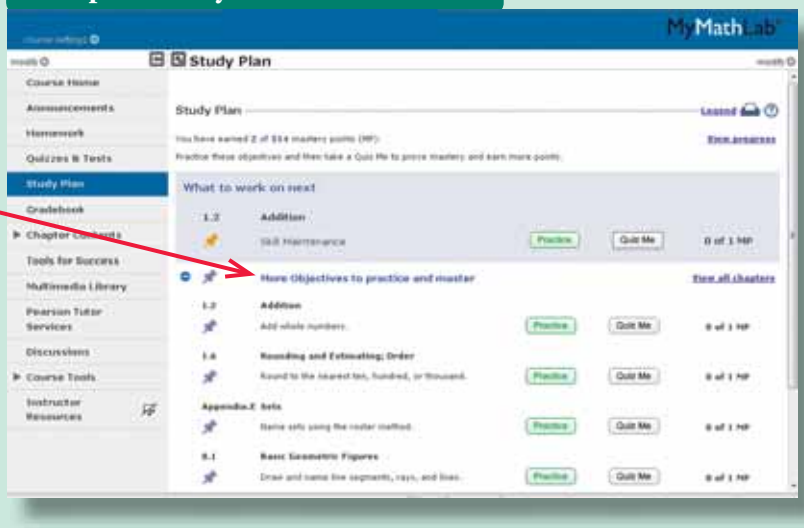
Personalized Homework



To help students achieve mastery, MyMathLab can generate **personalized homework** based on individual performance on tests or quizzes. Personalized homework allows students to focus on topics they have not yet mastered.

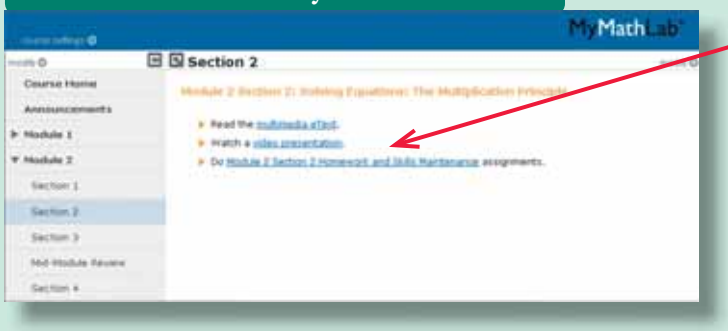
The **Adaptive Study Plan** makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content—reinforcing concepts that target each student's strengths and weaknesses.

Adaptive Study Plan



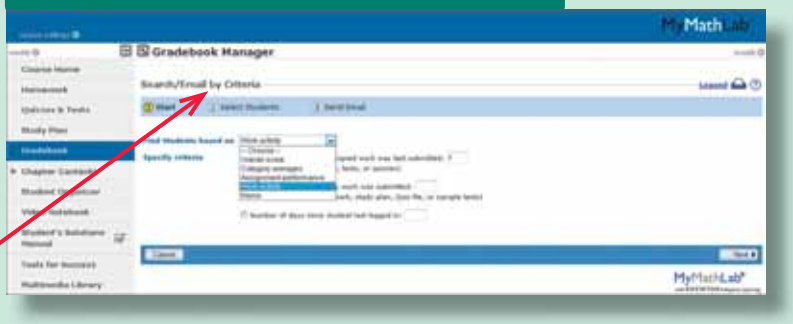
Flexible Design, Easy Start-Up, and Results for Instructors

Customized and Ready-to-Go Courses



Instructors can modify the site navigation and insert their own directions on course-level landing pages; also, a **custom MyMathLab** course can be built that reorganizes and structures the course material by chapters, modules, units—whatever the need may be.

Gradebook



The **comprehensive online gradebook** automatically tracks students' results on tests, quizzes, and homework and in the study plan. Instructors can use the gradebook to quickly intervene if students have trouble, or to provide positive feedback on a job well done. The data within MyMathLab are easily exported to a variety of spreadsheet programs, such as Microsoft Excel.® Instructors can determine which points of data to export and then analyze the results to determine success.

New features, such as **Search/Email by criteria**, make the gradebook a powerful tool for instructors. With this feature, instructors can easily communicate with both at-risk and successful students. They can search by score on specific assignments, noncompletion of assignments within a given time frame, last login date, or overall score.

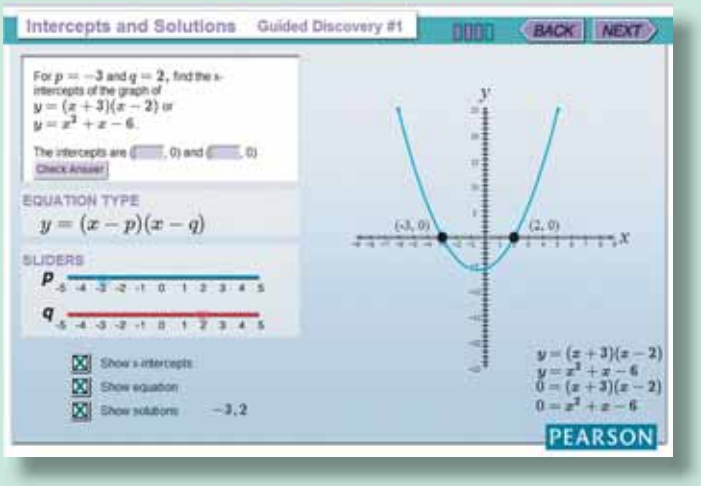
Special Bittinger Resources

in MyMathLab for Students and Instructors

In addition to robust course delivery, MyMathLab offers the full Bittinger eText, additional Bittinger Program features, and the entire set of instructor and student resources in one easy-to-access online location.

New! Active Learning Figures

In MyMathLab, Active Learning Figures guide students in exploring math concepts and reinforcing their understanding. Instructors can use Active Learning Figures in class or as media assignments in MyMathLab.



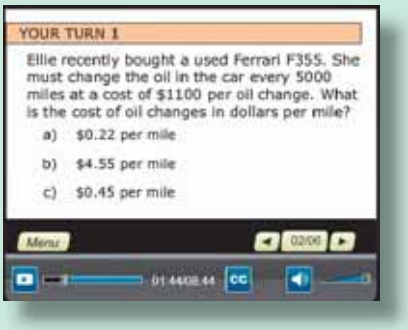
New! Integrated Bittinger Video Program and *MyMathGuide* workbook

Bittinger Video Program*

The Video Program is available in MyMathLab includes closed captioning and the following video types:

New! To-the-Point Objective Videos. These objective-based, interactive videos are incorporated into the Learning Path in MyMathLab and can be used along with the *MyMathGuide* workbook.

Chapter Test Prep Videos. The Chapter Test Prep Videos let students watch instructors work through step-by-step solutions to all the Chapter Test exercises from the textbook. Chapter Test Prep Videos are also available on YouTube (search using author name and book title).



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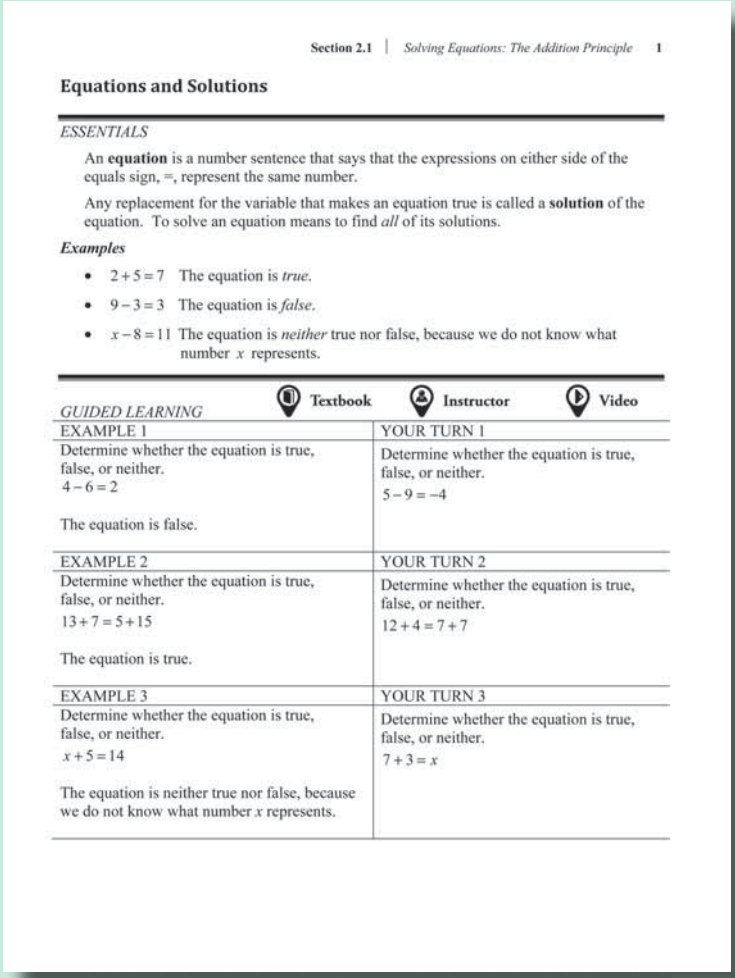
This objective-based workbook for guided, hands-on learning offers vocabulary, skill, and concept review—along with problem-solving practice—with space to show work and write notes. Incorporated in the Learning Path in MyMathLab, *MyMathGuide* can be used together with the To-the-Point Objective Video program, instructor lectures, and the textbook. Instructors can assign To-the-Point Objective Videos in MyMathLab in conjunction with the *MyMathGuide* workbook.



Section 2.1 | *Solving Equations: The Addition Principle* 1

Equations and Solutions

ESSENTIALS

An **equation** is a number sentence that says that the expressions on either side of the equals sign, =, represent the same number.

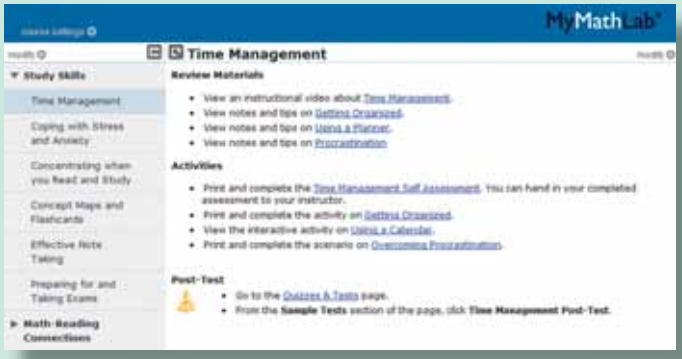
Any replacement for the variable that makes an equation true is called a **solution** of the equation. To solve an equation means to find *all* of its solutions.

Examples

- $2+5=7$ The equation is *true*.
- $9-3=3$ The equation is *false*.
- $x-8=11$ The equation is *neither* true nor false, because we do not know what number x represents.

GUIDED LEARNING Textbook Instructor Video

EXAMPLE 1	YOUR TURN 1
Determine whether the equation is true, false, or neither. $4-6=2$ The equation is false.	Determine whether the equation is true, false, or neither. $5-9=-4$
EXAMPLE 2	**YOUR TURN 2**
Determine whether the equation is true, false, or neither. $13+7=5+15$ The equation is true.	Determine whether the equation is true, false, or neither. $12+4=7+7$
EXAMPLE 3	**YOUR TURN 3**
Determine whether the equation is true, false, or neither. $x+5=14$ The equation is neither true nor false, because we do not know what number x represents.	Determine whether the equation is true, false, or neither. $7+3=x$



Study Skills Modules

In MyMathLab, interactive modules address common areas of weakness, including time-management, test-taking, and note-taking skills. Additional modules support career readiness. Instructors can assign module material with a post-quiz.

Additional Resources in MyMathLab

For Students

Student's Solutions Manual

By Judy Penna

Contains completely worked-out annotated solutions for all the odd-numbered exercises in the text. Also includes fully worked-out annotated solutions for all the exercises (odd- and even-numbered) in the Mid-Chapter Reviews, the Summary and Reviews, the Chapter Tests, and the Cumulative Reviews.

For Instructors

Instructor's Resource Manual with Tests and Mini Lectures*

(download only)
By Laurie Hurley

This manual includes resources designed to help both new and experienced instructors with course preparation and classroom management. This includes chapter-by-chapter teaching tips and support for media supplements. Contains two multiple-choice tests per chapter, six free-response tests per chapter, and eight final exams.

Instructor's Solutions Manual*

(download only)
By Judy Penna

This manual contains detailed, worked-out solutions to all odd-numbered exercises and brief solutions to the even-numbered exercises in the exercise sets.

PowerPoint® Lecture Slides*

(download only)

Present key concepts and definitions from the text.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit www.mymathlab.com or contact your Pearson representative.

*Also available in print or for download from the Instructor Resource Center (IRC) on http://www.pearsonglobaleditions.com/Bittinger

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CHAPTER

R

Review of Basic Algebra

PART 1 OPERATIONS

R.1 The Set of Real Numbers

R.2 Operations with Real Numbers

R.3 Exponential Notation and Order of Operations

PART 2 MANIPULATIONS

R.4 Introduction to Algebraic Expressions

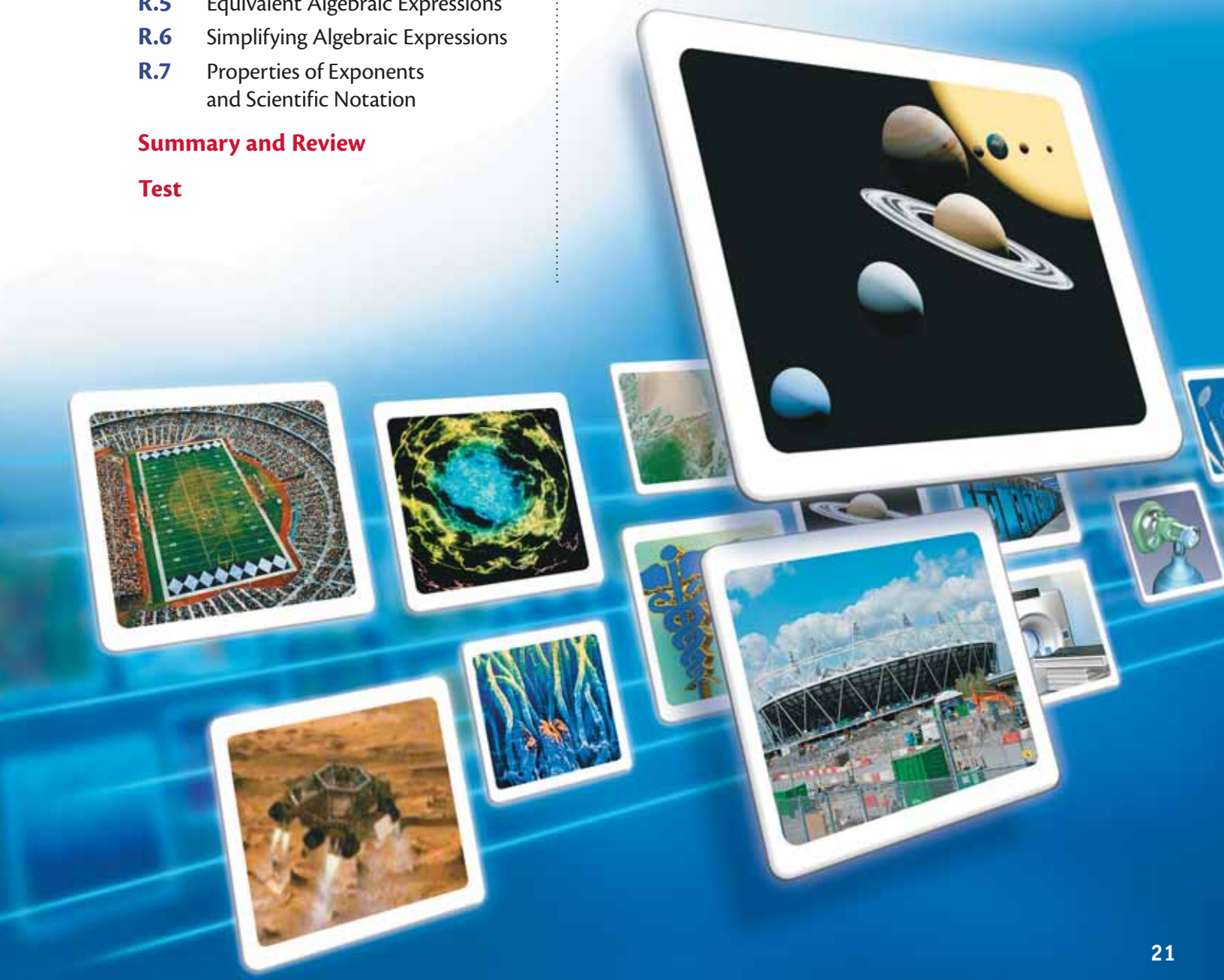
R.5 Equivalent Algebraic Expressions

R.6 Simplifying Algebraic Expressions

R.7 Properties of Exponents and Scientific Notation

Summary and Review

Test



STUDYING FOR SUCCESS *The Importance of Review*

- ☐ Continual review and practice sharpen skills and solidify concepts.
- ☐ Review helps you connect new material to math that you have previously studied.
- ☐ Review helps you pinpoint areas in which you need further work.

R.1

PART 1 OPERATIONS
The Set of Real Numbers

OBJECTIVES

- **a** Use roster notation and set-builder notation to name sets, and distinguish among various kinds of real numbers.
- **b** Determine which of two real numbers is greater and indicate which, using $<$ and $>$; given an inequality like $a < b$, write another inequality with the same meaning; and determine whether an inequality like $-2 \leq 3$ or $4 > 5$ is true.
- **c** Graph inequalities on the number line.
- **d** Find the absolute value of a real number.

a SET NOTATION AND THE SET OF REAL NUMBERS

A **set** is a collection of objects. In mathematics, we usually consider sets of numbers, such as the set of **real numbers**. There is a real number for every point on the real-number line. A **subset** is a set contained within another set. We begin by examining some subsets of the set of real numbers.

The set containing the numbers -5, 0, and 3 can be named $\{-5, 0, 3\}$. It is described using the **roster method**, which lists all members of a set. We use the roster method to describe three frequently used subsets of real numbers. Note that three dots are used to indicate that the pattern continues without end.

NATURAL NUMBERS, WHOLE NUMBERS, AND INTEGERS

Natural numbers are those numbers used for counting: $\{1, 2, 3, \ldots\}$.

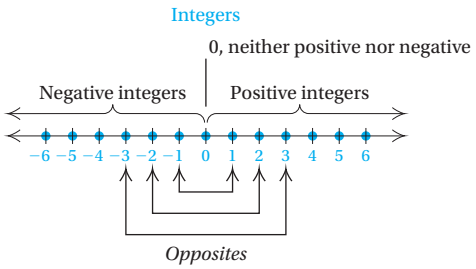
Whole numbers are the set of natural numbers with 0 included: $\{0, 1, 2, 3, \ldots\}$.

Integers are the set of whole numbers and their opposites:

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}.$$

Natural numbers are also called **counting numbers**.

The integers can be illustrated on the real-number line as follows.



The set of integers extends infinitely to the left and to the right of 0. The **opposite** of a number is found by reflecting it across the number 0. Thus the opposite of 3 is -3. The opposite of -4 is 4. The opposite of 0 is 0. We read a symbol like -3 as either "the opposite of 3" or "negative 3."

The natural numbers are called **positive integers**. The opposites of the natural numbers (those to the left of 0) are called **negative integers**. Zero is neither positive nor negative.

Do Exercises 1–3 (in the margin at right). ▶

Other subsets of real numbers are described using **set-builder notation**. With this notation, instead of listing all members of a set, we specify conditions under which a number is in a set. For example, the set of all odd natural numbers less than 9 can be described and read as follows:

$$\{x \mid x \text{ is an odd number less than } 9\}.$$

The set of all x such that x is an odd number less than 9

Using roster notation, we can write this set as $\{1, 3, 5, 7\}$.

EXAMPLE 1 Name the set consisting of the first six even whole numbers using both roster notation and set-builder notation.

Roster notation: $\{0, 2, 4, 6, 8, 10\}$

Set-builder notation: $\{x \mid x \text{ is one of the first six even whole numbers}\}$

Do Exercise 4. ▶

We can now describe the set of **rational numbers**.

RATIONAL NUMBERS

A **rational number** can be expressed as an integer divided by a nonzero integer. The set of rational numbers is

$$\left\{\frac{p}{q} \,\middle|\, p \text{ is an integer, } q \text{ is an integer, and } q \neq 0\right\}.$$

Rational numbers are numbers whose decimal representation either terminates or has a repeating block of digits.

The following are examples of rational numbers:

$$\frac{5}{8}, \quad \frac{12}{-7}, \quad \frac{-17}{15}, \quad -\frac{9}{7}, \quad \frac{39}{1}, \quad \frac{0}{6}.$$

Note that $\frac{39}{1} = 39$. Thus the set of rational numbers contains the integers. Using long division, we can write a fraction in decimal notation:

$$\frac{5}{8} = \underbrace{0.625}_{\text{Terminating}} \quad \text{and} \quad \frac{6}{11} = \underbrace{0.545454\ldots}_{\text{Repeating}} = 0.\overline{54}.$$

The bar in $0.\overline{54}$ indicates the repeating block of digits in decimal notation.

Do Exercises 5 and 6. ▶

Find the opposite of each number.

1. 9

2. -6

3. 0

4. Name the set consisting of the first seven odd whole numbers using both roster notation and set-builder notation.

Convert each fraction to decimal notation by long division and determine whether it is terminating or repeating.

5. $\frac{11}{16}$

GS **6.** $\frac{14}{3}$

$$3\overline{)14.0}$$

$$\begin{array}{r} \square.6 \\ 3\overline{)14.0} \\ \underline{12} \\ 20 \\ \underline{18} \\ \square \end{array}$$

← The remainder repeats.

Thus, $\frac{14}{3} = \square.\overline{6}$. The decimal notation is $\square$. (terminating/repeating)

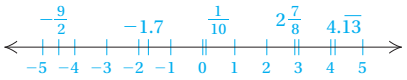
Answers

1. -9 **2.** 6 **3.** 0 **4.** $\{1, 3, 5, 7, 9, 11, 13\}$; $\{x \mid x \text{ is one of the first seven odd whole numbers}\}$ **5.** 0.6875; terminating **6.** $4.\overline{6}$; repeating

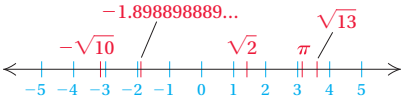
Guided Solution:

6. 4, 2, 4, repeating

The real-number line has a point for every rational number.



However, there are many points on the line for which there is no rational number. These points correspond to what are called **irrational numbers**.

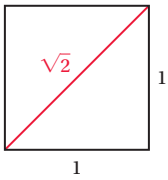


Numbers like π, $\sqrt{2}$, $-\sqrt{10}$, $\sqrt{13}$, and $-1.898898889\ldots$ are examples of irrational numbers. The decimal notation for an irrational number *neither* terminates *nor* repeats. Recall that decimal notation for rational numbers either terminates or has a repeating block of digits.

IRRATIONAL NUMBERS

Irrational numbers are numbers whose decimal representation neither terminates nor has a repeating block of digits. They cannot be represented as the quotient of two integers.

The irrational number $\sqrt{2}$ (read "the square root of 2") is the length of the diagonal of a square with sides of length 1. It is also the number that, when multiplied by itself, gives 2. No rational number can be multiplied by itself to get 2, although some approximations come close:



1.4 is an *approximation* of $\sqrt{2}$ because
$(1.4)^2 = (1.4)(1.4) = 1.96$;

1.41 is a better approximation because
$(1.41)^2 = (1.41)(1.41) = 1.9881$;

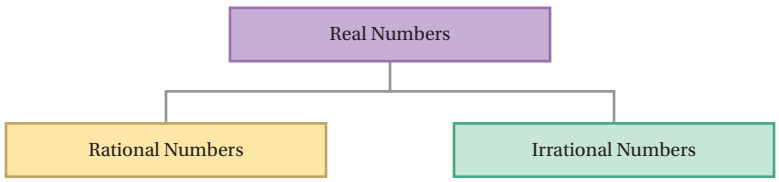
1.4142 is an even better approximation because
$(1.4142)^2 = (1.4142)(1.4142) = 1.99996164$.

We say that 1.4142 is a rational approximation of $\sqrt{2}$ because

$$(1.4142)^2 = 1.99996164 \approx 2.$$

The symbol $\approx$ means "is approximately equal to." We can find rational approximations for square roots and other irrational numbers using a calculator.

The set of all rational numbers, combined with the set of all irrational numbers, gives us the set of **real numbers**.

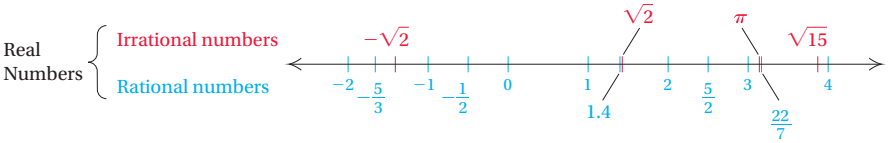


REAL NUMBERS

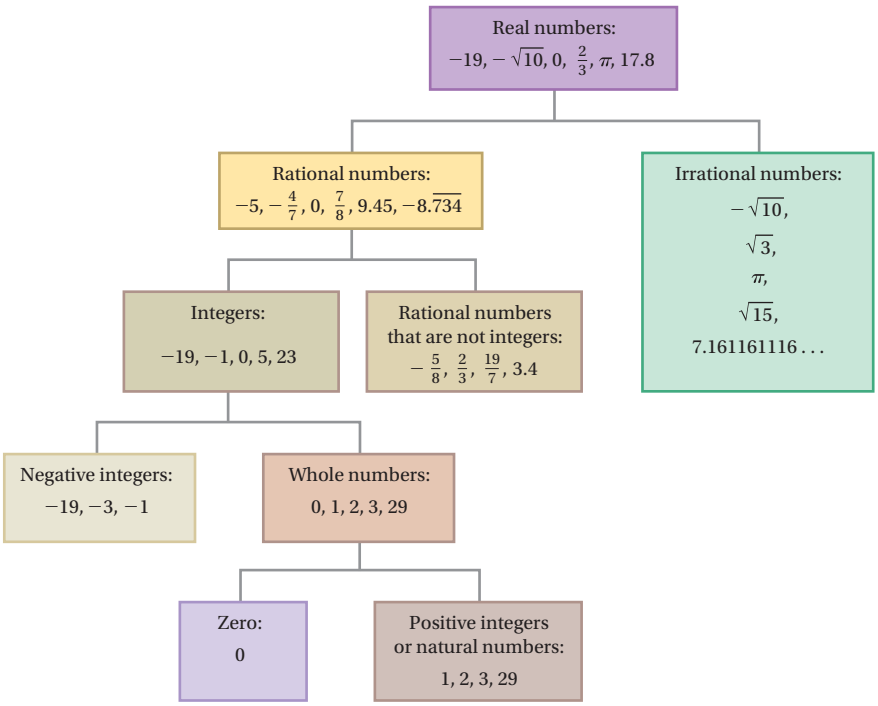
The set of **real numbers** is

$$\{x \mid x \text{ is a rational number } \textit{or } x \text{ is an irrational number}\}.$$

Every point on the number line represents some real number and every real number is represented by some point on the number line.



The following figure shows the relationships among various sets of real numbers.



Do Exercise 7. ▶

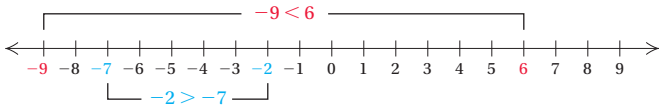
7. Given the numbers

$$20,\ -10,\ -5.34,\ 18.999,\ \frac{11}{45},\ \sqrt{7},\ -\sqrt{2},\ \sqrt{16},\ 0,\ -\frac{2}{3},\ 9.34334333433334\ldots:$$

a) Name the natural numbers.
b) Name the whole numbers.
c) Name the integers.
d) Name the irrational numbers.
e) Name the rational numbers.
f) Name the real numbers.

b ORDER FOR THE REAL NUMBERS

Real numbers are named in order on the number line, with larger numbers named further to the right. For any two numbers on the line, the one to the left is less than the one to the right.



We use the symbol $<$ to mean "**is less than.**" The sentence $-9 < 6$ means "-9 is less than 6." The symbol $>$ means "**is greater than.**" The sentence $-2 > -7$ means "-2 is greater than -7." Sentences containing $<$ or $>$ are called **inequalities**.

Answer

7. (a) 20, $\sqrt{16}$; **(b)** 20, $\sqrt{16}$, 0; **(c)** 20, -10, $\sqrt{16}$, 0; **(d)** $\sqrt{7}$, $-\sqrt{2}$, 9.34334333433334 . . . ; **(e)** 20, -10, -5.34, 18.999, $\frac{11}{45}$, $\sqrt{16}$, 0, $-\frac{2}{3}$; **(f)** 20, -10, -5.34, 18.999, $\frac{11}{45}$, $\sqrt{7}$, $-\sqrt{2}$, $\sqrt{16}$, 0, $-\frac{2}{3}$, 9.34334333433334 . . .

Insert $<$ or $>$ for $\square$ to write a true sentence.

8. $-5 \square -4$

9. $-\frac{1}{4} \square -\frac{1}{2}$

10. $87 \square 67$

11. $-9.8 \square -4\frac{2}{3}$

12. $6.78 \square -6.77$

13. $-\frac{4}{5} \square -0.86$

14. $\frac{14}{29} \square \frac{17}{32}$

15. $-\frac{12}{13} \square -\frac{14}{15}$

16. $1.8 \square 1.08$

Write a different inequality with the same meaning.

17. $x > 6$

18. $-4 < 7$

Determine whether each of the following is true or false.

19. $6 \geq -9.4$

20. $-18 \leq -18$

21. $-7.6 \leq -10\frac{4}{5}$

22. $-\frac{24}{27} \geq -\frac{25}{28}$

Answers

8. $<$ **9.** $>$ **10.** $>$ **11.** $<$ **12.** $>$
13. $>$ **14.** $<$ **15.** $>$ **16.** $>$ **17.** $6 < x$
18. $7 > -4$ **19.** True **20.** True
21. False **22.** True

EXAMPLES Use either $<$ or $>$ for $\square$ to write a true sentence.

2. $4 \square 9$ Since 4 is to the left of 9, 4 is less than 9, so $4 < 9$.

3. $-8 \square 3$ Since -8 is to the left of 3, we have $-8 < 3$.

4. $7 \square -12$ Since 7 is to the right of -12, then $7 > -12$.

5. $-21 \square -5$ Since -21 is to the left of -5, we have $-21 < -5$.

6. $4.79 \square 4.97$ Since 4.79 is to the left of 4.97, we have $4.79 < 4.97$.

7. $-2.7 \square -\frac{3}{2}$ Since $-\frac{3}{2} = -1.5$ and -2.7 is to the left of -1.5, we have $-2.7 < -\frac{3}{2}$.

8. $\frac{5}{8} \square \frac{7}{11}$ We convert to decimal notation $\left(\frac{5}{8} = 0.625 \text{ and } \frac{7}{11} = 0.6363\ldots\right)$ and compare. Thus, $\frac{5}{8} < \frac{7}{11}$.

◀ **Do Exercises 8–16.**

All positive real numbers are greater than zero and all negative real numbers are less than zero.

If x is a positive real number, then $x > 0$.
If x is a negative real number, then $x < 0$.

Note that $-8 < 5$ and $5 > -8$ are both true. Every true inequality yields another true inequality if we interchange the numbers or variables and reverse the direction of the inequality sign.

$a < b$ also has the meaning $b > a$.

EXAMPLES Write a different inequality with the same meaning.

9. $a < -5$ The inequality $-5 > a$ has the same meaning.

10. $-3 > -8$ The inequality $-8 < -3$ has the same meaning.

◀ **Do Exercises 17 and 18.**

Expressions like $a \leq b$ and $b \geq a$ are also **inequalities**. We read $\boldsymbol{a \leq b}$ as "***a* is less than or equal to *b*.**" We read $\boldsymbol{a \geq b}$ as "***a* is greater than or equal to *b*.**" If a is nonnegative, then $a \geq 0$.

EXAMPLES Determine whether each of the following is true or false.

11. $-8 \leq 5.7$ True since $-8 < 5.7$ is true.

12. $-8 \leq -8$ True since $-8 = -8$ is true.

13. $-7 \geq 4\frac{1}{3}$ False since neither $-7 > 4\frac{1}{3}$ nor $-7 = 4\frac{1}{3}$ is true.

14. $-\frac{2}{3} \geq -\frac{5}{4}$ True since $-\frac{2}{3} = -0.666\ldots$ and $-\frac{5}{4} = -1.25$ and $-0.666\ldots > -1.25$.

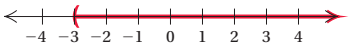
◀ **Do Exercises 19–22.**

c GRAPHING INEQUALITIES ON THE NUMBER LINE

A replacement that makes an inequality true is called a **solution**. The set of all solutions is called the **solution set**. A **graph** of an inequality is a drawing that represents its solution set.

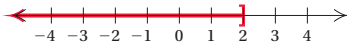
EXAMPLE 15 Graph the inequality $x > -3$ on the number line.

The solutions consist of all real numbers greater than -3, so we shade all numbers greater than -3. Since -3 is not a solution, we use a parenthesis at -3. The graph represents the solution set $\{x|x > -3\}$.



EXAMPLE 16 Graph the inequality $x \leq 2$ on the number line.

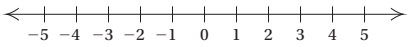
We make a drawing that represents the solution set $\{x|x \leq 2\}$. The graph consists of 2 as well as the numbers less than 2. We shade all numbers to the left of 2 and use a bracket at 2 to indicate that it is also a solution.



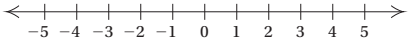
Do Exercises 23–26.

Graph each inequality.

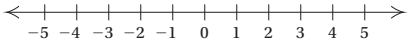
23. $x > -1$



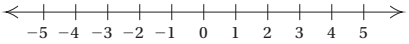
24. $x \leq 5$



25. $0 < x$

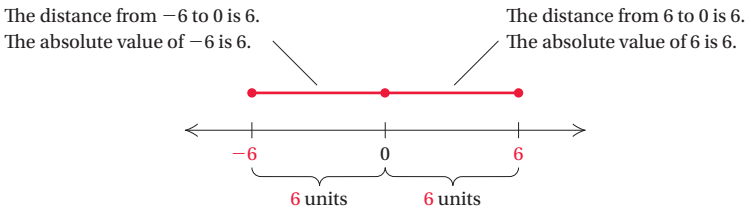


26. $-\frac{5}{2} \geq x$



d ABSOLUTE VALUE

We call the distance of a number from 0 on the number line the **absolute value** of the number. Since distance is always a nonnegative number, the absolute value of a number is always greater than or equal to 0.



ABSOLUTE VALUE

The **absolute value** of a number is its distance from 0 on the number line. We use the symbol $|x|$ to represent the absolute value of a number x.

EXAMPLES Find the absolute value.

17. $|-7|$ The distance of -7 from 0 is 7, so $|-7|$ is 7.

18. $|12|$ The distance of 12 from 0 is 12, so $|12|$ is 12.

19. $|0|$ The distance of 0 from 0 is 0, so $|0|$ is 0.

20. $\left|\frac{4}{5}\right| = \frac{4}{5}$

21. $|-3.86| = 3.86$

Do Exercises 27–30.

Find the absolute value.

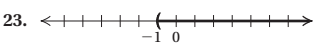
GS **27.** $|2|$

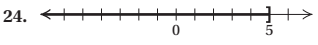
The distance of 2 from 0 is ____, so $|2|$ is ____.

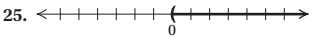
28. $\left|-\frac{1}{4}\right|$ **29.** $\left|\frac{3}{2}\right|$

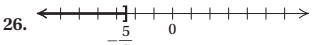
30. $|-2.3|$

Answers

23.


24.


25.


26.


27. 2 **28.** $\frac{1}{4}$ **29.** $\frac{3}{2}$ **30.** 2.3

Guided Solution:

27. 2, 2

R.1 Exercise Set

For Extra Help MyMathLab® MathXL® PRACTICE WATCH READ REVIEW

Reading Check

Choose from the column on the right the set of numbers that matches the description.

RC1. ____ Natural numbers

RC2. ____ Whole numbers

RC3. ____ Integers

RC4. ____ Rational numbers

RC5. ____ Irrational numbers

RC6. ____ Real numbers

a) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

b) $\{0, 1, 2, 3, \ldots\}$

c) $\{1, 2, 3, 4, \ldots\}$

d) $\{x \mid x \text{ is a rational number } \textit{or } x \text{ is an irrational number}\}$

e) $\{x \mid x \text{ cannot be represented as the quotient of two integers}\}$

f) $\left\{\dfrac{p}{q} \,\middle|\, p \text{ is an integer, } q \text{ is an integer, and } q \neq 0\right\}$

a Given the numbers $-6, 0, 1, -\frac{1}{2}, -4, \frac{7}{9}, 12, -\frac{6}{5}, 3.45, 5\frac{1}{2}, \sqrt{3}, \sqrt{25}, -\frac{12}{3}, 0.131331333133331\ldots$:

1. Name the natural numbers.

2. Name the whole numbers.

3. Name the rational numbers.

4. Name the integers.

5. Name the real numbers.

6. Name the irrational numbers.

Given the numbers $-\sqrt{5}, -3.43, -11, 12, 0, \frac{11}{34}, -\frac{7}{13}, \pi, -3.565665666566665\ldots$:

7. Name the whole numbers.

8. Name the natural numbers.

9. Name the integers.

10. Name the rational numbers.

11. Name the irrational numbers.

12. Name the real numbers.

Use roster notation to name each set.

13. The set of all letters in the word "math"

14. The set of all letters in the word "solve"

15. The set of all positive integers less than 13

16. The set of all odd whole numbers less than 13

17. The set of all even natural numbers

18. The set of all negative integers greater than -4

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Use set-builder notation to name each set.

19. $\{0, 1, 2, 3, 4, 5\}$

20. $\{4, 5, 6, 7, 8, 9, 10\}$

21. The set of all rational numbers

22. The set of all real numbers

23. The set of all real numbers greater than -3

24. The set of all real numbers less than or equal to 21

b Use either $<$ or $>$ for $\square$ to write a true sentence.

25. $13 \square 0$

26. $18 \square 0$

27. $-8 \square 2$

28. $7 \square -7$

29. $-8 \square 8$

30. $0 \square -11$

31. $-8 \square -3$

32. $-6 \square -3$

33. $-2 \square -12$

34. $-7 \square -10$

35. $-9.9 \square -2.2$

36. $-13\frac{1}{5} \square \frac{11}{250}$

37. $37\frac{1}{5} \square -1\frac{67}{100}$

38. $-13.99 \square -8.45$

39. $\frac{6}{13} \square \frac{13}{25}$

40. $-\frac{14}{15} \square -\frac{27}{53}$

Write a different inequality with the same meaning.

41. $-8 > x$

42. $x < 7$

43. $-12.7 \leq y$

44. $10\frac{2}{3} \geq t$

Write true or false.

45. $6 \leq -6$

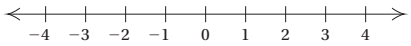
46. $-7 \leq -7$

47. $5 \geq -8.4$

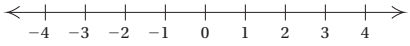
48. $-11 \geq -13\frac{1}{2}$

c Graph each inequality.

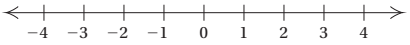
49. $x < -2$



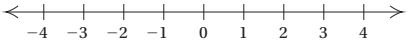
50. $x < -1$



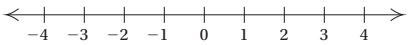
51. $x \leq -2$



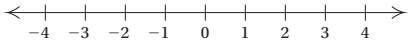
52. $x \geq -1$



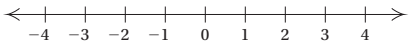
53. $x > -3.3$



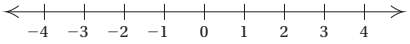
54. $x < 0$



55. $x \geq 2$



56. $x \leq 0$



d Find the absolute value.

57. $|-6|$ **58.** $|-3|$ **59.** $|28|$ **60.** $|16|$ **61.** $|-35|$

62. $|-127|$ **63.** $\left|-\frac{2}{3}\right|$ **64.** $\left|-\frac{13}{8}\right|$ **65.** $|42.8|$ **66.** $|16.4|$

67. $|986|$ **68.** $|465|$ **69.** $\left|\frac{0}{-7}\right|$ **70.** $\left|\frac{0}{-15}\right|$

Synthesis

To the student and the instructor: The Synthesis exercises found at the end of every exercise set challenge students to combine concepts or skills studied in that section or in preceding parts of the text.

Use either $\leq$ or $\geq$ for $\square$ to write a true sentence.

71. $|-3| \square 5$ **72.** $|-5| \square |-2|$ **73.** $|4| \square |-7|$ **74.** $|-8| \square |8|$

75. List the following numbers in order from least to greatest.

$$\frac{1}{11},\ 1.1\%,\ \frac{2}{7},\ 0.3\%,\ 0.11,\ \frac{1}{8}\%,\ 0.009,\ \frac{99}{1000},\ 0.286,\ \frac{1}{8},\ 1\%,\ \frac{9}{100}$$

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